Properties of the magnetospheric backward wave oscillator inferred from CLUSTER measurements of VLF chorus elements

E Titova, A Demekhov, B Kozelov, O Santolík, E Macúšová, Jean-Louis Rauch, Jean-Gabriel Trotignon, D Gurnett, J Pickett

Properties of the magnetospheric backward wave oscillator inferred from CLUSTER measurements of VLF chorus elements

E. Titova,[1,2] A. Demekhov,[3] B. Kozelov,[1] O. Santolík,[4] E. Macúšová,[4] J.-L. Rauch,[5] J.-G. Trotignon,[5] D. Gurnett,[6] and J. Pickett[6]

Received 13 March 2012; revised 22 June 2012; accepted 25 June 2012; published 8 August 2012.

[1] According to the backward wave oscillator (BWO) model, a sharp gradient (or step-like deformation) on the electron distribution function is the most important factor in chorus generation, but such a feature is very difficult to observe directly. The properties of the step in the BWO model determine the dimensionless parameter q quantifying the excess of the energetic electron flux above the absolute-instability threshold. This parameter, in turn, is related to the frequency sweep rate of chorus elements, which we obtained by using data from the WBD instrument onboard the CLUSTER satellites in the equatorial region for more than 7000 chorus elements. Then, using the CLUSTER data for the plasma density and magnetic field, we calculated q assuming the validity of the BWO theory and found that the q values depend only weakly on the density; the average values of $q \approx 7$ for the lower band chorus ($f/f_{ce} < 0.5$) and $q \approx 13$ for the upper band ($f/f_{ce} > 0.5$). These q values constitute a large excess over the generation threshold (q > 3) resulting from numerical simulation of discrete elements with rising frequency and are thus consistent with the simulations. Another important feature of the q parameter is the significant scatter of its values during each Cluster passage of the generation region. Using the obtained q values we estimate the relative height of the step in the electron distribution function to lie in the range from 0.01 to 0.3.

Citation: Titova, E., A. Demekhov, B. Kozelov, O. Santolík, E. Macúšová, J.-L. Rauch, J.-G. Trotignon, D. Gurnett, and J. Pickett (2012), Properties of the magnetospheric backward wave oscillator inferred from CLUSTER measurements of VLF chorus elements, *J. Geophys. Res.*, *117*, A08210, doi:10.1029/2012JA017713.

1. Introduction

[2] VLF chorus emissions are observed as a succession of repeating discrete elements, usually with rising frequency. Typical frequencies of these emissions are ∼(0.5–5) kHz, duration of their elements ∼(0.1–0.5) s, and frequency sweep rate of these chorus elements $df/dt \sim$ (1–10) kHz/s. VLF chorus emissions are the most intense and frequently observed VLF signals in the Earth's magnetosphere [*Parrot et al.*, 2004; *Santolík et al.*, 2003, 2004]. Recently VLF chorus has attracted special attention primarily due to the new observations from the CLUSTER and THEMIS satellites, which enable more a detailed comparison of the observed chorus features with the theory and numerical simulations of their generation, and a renewed emphasis on the physics of the radiation belts. The role of chorus emissions in the acceleration of electrons in the radiation belts to relativistic energies are discussed very actively in the literature [*Horne and Thorne*, 1998; *Summers and Ma*, 2000; *Meredith et al.*, 2003; *Demekhov et al.*, 2009]. A hypothesis of the origin of plasmaspheric hiss from chorus emissions was suggested recently by *Parrot et al.* [2004], *Santolík et al.* [2006], and *Bortnik et al.* [2008].

[3] It is accepted that the chorus emissions are generated by the cyclotron instability of ring current electrons with energies of a few tens of keV, and the source is located near the equatorial region in the morning and night sectors (at daytime chorus is also generated in high-latitude minimum B pockets [see, e.g., *Tsurutani and Smith*, 1977]). However, a smooth distribution function of energetic electrons with reasonable flux and anisotropy does not ensure the necessary amplification of waves (in the linear approach) at L shells 4 to 5 where the discrete VLF chorus elements are frequently observed. To solve this problem *Trakhtengerts* [1995, 1999] proposed a new generation regime of cyclotron instability in the magnetosphere for which a sharp gradient (step-like deformation) on the distribution function of energetic electrons is necessary. This is the backward wave oscillator (BWO) regime of VLF chorus generation which is

[1]Polar Geophysical Institute, Apatity, Russia.
[2]Space Research Institute, Moscow, Russia.
[3]Institute of Applied Physics, Nizhny Novgorod, Russia.
[4]IAP/CAS, Prague, Czech Republic.
[5]LPCE/CNRS, Orleans, France.
[6]Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa, USA.

Corresponding author: E. Titova, Polar Geophysical Institute, Fersmana 14, 184209, Apatity, Russia. (lena.titova@gmail.com)

0148-0227/12/2012JA017713

similar to the backward wave oscillator in laboratory electronic devices [*Ginzburg and Kuznetsov*, 1981]. The step-like deformation of the distribution function in the velocity space can arise naturally due to cyclotron interactions of energetic electrons with ELF/VLF noise-like emissions having an upper-frequency cutoff. The step is situated at the boundary between resonant and nonresonant particles associated with this frequency cutoff and is formed because resonant particles are subject to pitch angle diffusion and loss, while nonresonant particles preserve their velocity distribution (see *Trakhtengerts et al.* [1996] for more details). According to the BWO model, the step-like deformation yields the large growth rate of the whistler mode waves and development of the absolute cyclotron instability in a narrow region near the equatorial plane. The nonlinear stage of the instability produces a succession of discrete signals with an increasing frequency within each element.

[4] Recently chorus elements have been obtained in numerical simulations with an initially smooth velocity distribution of energetic electrons [e.g., *Hikishima et al.*, 2009]. These results in most respects confirmed the main ideas of the BWO model with regard to the nonlinear stage of the chorus generation, i.e., similarly to analytical results of *Omura et al.* [2008], they supported the role of nonlinear trapping in the frequency drift formation. Moreover, these simulations were performed with very high flux and/or anisotropy of energetic electrons which have never been observed by CLUSTER in the chorus generation region and, in this way, they support the necessity of an initial sharp velocity space gradient for the chorus formation.

[5] It is clear that a sharp gradient (or a step-like deformation) of the electron distribution function is the most important parameter of the BWO model. However, experimental verification of the existence of step-like features in the velocity space is a difficult problem. Such features of the distribution functions have not been detected directly because of the time resolution and pitch angle resolution of existing instrumentation are restricted. Indeed, to detect a step-like feature, an energetic-particle detector with a good resolution in both energy W_e and pitch angle θ are necessary. The desired resolution is $\delta\theta \sim \delta W_e \sim$ 1–10% for electrons with energies 1 keV < W_e < 50 keV [*Demekhov et al.*, 2003]. Moreover, sharp velocity space gradients of the distribution function ensure its strong spatial inhomogeneity. The characteristic scale of such an inhomogeneity along the magnetic field is of the order of 100 km.

[6] However, the properties of the step in the BWO model determine the dimensionless parameter q which characterizes the excess of an energetic electron flux over a threshold of the BWO regime (see equation (8) below). This parameter, in turn, determines the frequency sweep rate of chorus elements [*Trakhtengerts et al.*, 2004]. Thus, by measuring the frequency sweep rate in the source region we can obtain the q parameter values and analyze the properties of a step on the distribution function, assuming the validity of the BWO model. A decrease of the mean values of the frequency sweep rate with increasing mean cold plasma density was demonstrated by *Macúšová et al.* [2010]. This result is in a reasonable agreement with the BWO model if the q parameter is independent of the plasma density. In this paper, we determine the values of the q parameter in the magnetospheric BWO model for individual chorus elements on the basis of multi-instrument CLUSTER satellite measurements. Then we compare q values, obtained from observations, with results of numerical simulations of the BWO model, estimate the relative height of the step on the electron distribution function and discuss the spread of the q parameter values. In the concluding sections we discuss and summarize our results.

2. Theoretical Background

[7] The expression for the frequency sweep rate of chorus elements df/dt can be written as [*Trakhtengerts*, 1999; *Trakhtengerts et al.*, 2004]:

$$df/dt = \Omega_{tr}^2/(2\pi)^2 \tag{1}$$

where the trapping frequency $\Omega_{tr} = (k\, \upsilon_{perp}\, \omega_{ce}\, b_w)^{1/2}$, $b_w = B_\sim/B_L$ is the ratio of the whistler wave magnetic field amplitude $B_\sim$ to the geomagnetic field B_L, υ_{perp} is the electron velocity component across the geomagnetic field, k is the parallel wave number, and ω_{ce} is the electron gyrofrequency. Equation (1) takes into account the nonlinear frequency shift due to the sideband instability resulting from particle trapping in the wavefield but neglects the inhomogeneity effects [*Helliwell*, 1967; *Trakhtengerts*, 1999]. Earlier estimates based on Magion 5 data [*Titova et al.*, 2003] showed that the inhomogeneity contribution was less important for high activity cases. Note that equation (1) is consistent with the results of *Omura et al.* [2008] who calculated the frequency drift rate by maximizing the nonlinear growth rate for the electron distribution with a plateau hole in the trapping region in the velocity space. It is not just a coincidence since flattening of the distribution in the trapping region and the sideband instability which was put forward as the basis of equation (1) are closely interrelated.

[8] In the case of the BWO regime and/or an absolute instability, the BWO growth rate γ_{BWO} relates to the trapping frequency Ω_{tr} [*Trakhtengerts*, 1995, 1999] as

$$\gamma_{BWO}/\Omega_{tr} \approx 3\pi/32 \tag{2}$$

Substituting equation (2) into equation (1) gives

$$df/dt = 0.3\gamma_{BWO}^2 \tag{3}$$

According to the BWO model, in the case of propagation along the magnetic field the growth rate is given by [*Trakhtengerts et al.*, 2004; *Bespalov and Demekhov*, 2009]

$$\gamma_{BWO} = \pi/2 \cdot \left(q^{1/2} - q^{-1/3}\right)/T_0 \tag{4}$$

Here, T_0 is the characteristic time scale of the BWO defined as

$$T_0 = l_{BWO}\left(1/V_g + 1/V_{st}\right) \tag{5}$$

where V_{st} is the parallel velocity of resonant electrons corresponding to the step (abbreviated as “st”) at the distribution function,

$$V_{st} = 2\pi(f_{ce} - f)/k \tag{6}$$

In equation (5) $l_{BWO} \approx 1.76\, (2\pi\, R_E^2 L^2/6k)^{1/3}$ is the length of the source region, V_g is the wave group velocity for which it is easily shown that V_g = 2 $V_{st}\, f/f_{ce}$ for parallel propagating whistler mode waves (see equation (6) for V_{st}), and assuming $f_p^2 \gg f\,(f - f_{ce})$, where f_p is the plasma frequency, and f and

f_{ce} are the wave frequency and electron gyrofrequency; L is the L-shell, R_E is the Earth's radius, f and f_{ce} are the wave frequency and electron gyrofrequency, and $q = S/S_{\rm thr}$ is the dimensionless parameter quantifying the excess of the energetic-electron flux S above the threshold $S_{\rm thr}$ for the BWO generation regime. More specifically, the q parameter was estimated in *Demekhov and Trakhtengerts* [2005, 2008] for the distribution function of energetic electrons F_0 with a step-like deformation in the parallel velocity component:

$$F_0 = N_h\left[1 - b + b\theta\left(V_{st} + \upsilon_\parallel\right)\right]F\left(\upsilon_\parallel, \upsilon_{perp}\right) \tag{7}$$

where N_h is the density of the energetic electrons, b *is* the relative height of the step, $\theta(x)$ is the Heaviside unit function, $\upsilon_\parallel$ and υ_{perp} are the parallel and perpendicular velocities with respect to the geomagnetic field, and F ($\upsilon_\parallel$, υ_{perp}) is the smooth part of the distribution function.

[9] Then q can be expressed [*Demekhov and Trakhtengerts*, 2005, 2008] as

$$q = \frac{2\pi^2 e^2}{mc^2} V_g t_0^2 b N_h \int v_\perp^3 F^* dv_\perp \tag{8}$$

where e, m, respectively are the charge and mass of the electron; $t_0 = l_{BWO}/V_{st}$, and F^* is the distribution function at $\upsilon_\parallel = -V_{st}$.

[10] As is seen from equation (8), the q parameter is proportional to the relative step height and the concentration of energetic electrons. The aim of this work is to determine the magnitude of this parameter for the magnetospheric BWO.

[11] Substituting equation (4) into equation (3) gives the following equation relating the q parameter to the frequency drift rate:

$$\left(q^{1/2} - q^{-1/3}\right)^2 = 3 \cdot (2/\pi)^2 \cdot df/dt \cdot T_0^2 \tag{9}$$

As it follows from equation (9), the q parameter is related to the frequency sweep rate and the characteristic BWO time scale T_0. Substituting in equation (5) the length of the source region l_{BWO}, the parallel velocity of resonant electrons V_{st} and the group velocity of the whistler mode waves V_g yields the formula

$$T_0 = \frac{L^{11/3}}{3 \cdot 10^{-2}} \cdot \frac{1 + 2x}{2x} \cdot \frac{x^{1/3}}{(1 - x)^{4/3}} \cdot N_e^{1/3} \tag{10}$$

where $x = f/f_{ce}$ is the chorus frequency normalized to the electron gyrofrequency, and N_e is the plasma density in $\rm cm^{-3}$. It is seen from equation (10) that the BWO time scale T_0 increases with L shell and plasma density N_e, and it is straightforward to determine that T_0 increases with frequency for $x > 0.215$.

[12] As it follows from equations (5) and (9), the determination of the parameter q requires the simultaneous measurements of the frequencies and sweep rates of chorus elements, magnetic fields and plasma density in the source region; the CLUSTER satellites provide us with all of these measurements.

3. Analysis of the *q* Parameter for the Magnetospheric BWO

3.1. The CLUSTER Spacecraft Observations

[13] Our analysis is based on simultaneous observations from several instruments onboard the CLUSTER spacecraft which measure the VLF waves (WBD), the total plasma density (WHISPER) and the magnetic field (FGM). To analyze the sweep rate of the chorus elements on the CLUSTER satellites we used the waveform data of the Wide Band Data (WBD) instrument [*Gurnett et al.*, 2001], which obtains spectrograms of one electric or one magnetic component with high time and frequency resolution. We use these spectrograms to determine the frequency and frequency sweep rate of discrete chorus elements. In this study we consider the chorus emissions both above and below one-half of the electron gyrofrequency and the discrete elements with rising frequency only.

[14] To find the q parameter for each chorus element from equation (9), we need the measurements of the magnetic field and plasma density with good temporal resolution. Therefore, special processing of the WHISPER active sounder [*Décréau et al.*, 2001] data, providing maximum temporal resolution, has been performed for 7 orbits. In these orbits the chorus emissions were observed during (date, UT time): Apr. 12, 2001, 04:35–04:57; Oct. 21, 2001, 23:15–23:35; March 25, 2002, 13:56–14:21; Apr. 18, 2002, 08:38–09:05; Dec. 6, 2003, 14:30–15:00; Feb. 6, 2004, 10:40–11:08; Feb. 8, 2005, 11:50–12:13.

[15] The relationship between the average values of the total plasma density for these events and the frequency sweep rate of the chorus elements was considered in *Macúšová et al.* [2010]. Since the repetition time scale of chorus elements is usually much less than the WHISPER time resolution (≥4s), we used a linear interpolation for the density within each 4-s interval.

[16] The values of the magnetic field and/or electron gyrofrequency are determined from measurements of the onboard fluxgate magnetometers (FGM) [*Balogh et al.*, 2001]. When the CLUSTER spacecraft move away from the equatorial plane the locally measured magnetic field was reduced by a factor obtained from the geometry of the magnetic field lines calculated from N. A. Tsyganenko and D. P. Stern's model (A new-generation global magnetosphere field model, based on spacecraft magnetometer data, ISTP Project Newsletter, *6*(1), 21, 1996). The equatorial magnetic field was estimated by multiplying the observed magnetic field by the ratio of the model field at the equator to that at the position of the spacecraft. Then we compared the values of the calculated equatorial half-gyrofrequency with the position of the gap in the chorus amplitude, and these were usually in good agreement.

[17] For these 7 orbits of the CLUSTER satellite we obtained the frequency sweep rates for about 7000 elements in the source region (within $\pm 5°$ near the magnetic equator). Note that the q parameter values can be quite large in the magnetospheric BWO, $q \approx$ 10–20 [*Demekhov and Trakhtengerts*, 2008; *Macúšová et al.*, 2010] (see also below), and for $q \gg 1$ the second term in parentheses in equation (9) can be ignored. However, we want to determine the q value without any a priori assumptions about its value. Therefore, equation (9) was solved numerically for each chorus element.

3.2. Values of the *q* Parameter for the Magnetospheric BWO and Their Relationship With Cold Plasma Density

[18] As it follows from equation (9), the experimentally determined q parameter depends on the BWO characteristic time scale T_0 and the frequency sweep rate. The time scale T_0 in turn depends on L shell, plasma density N_e and the

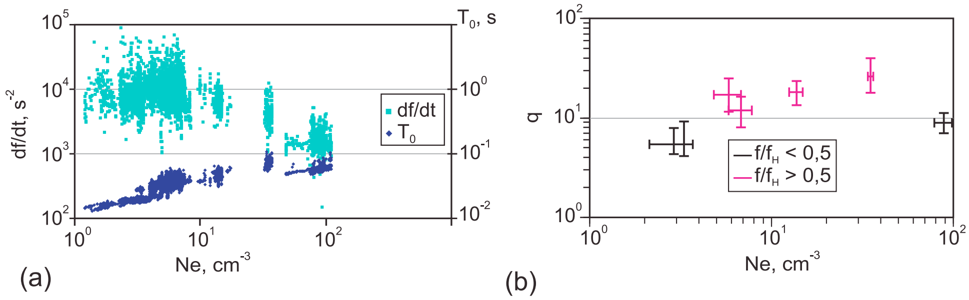

Figure 1. (a) The frequency sweep rates df/dt (cyan points) and the time scale of the BWO T_0 (blue points) depending on the plasma density for all the chorus elements measured on 7 orbits of the CLUSTER satellites. (b) The median the values of the q parameter for chorus elements with frequencies below (black) and above half (magenta) one-half of the electron gyrofrequency depending on the average cold plasma density for the 7 Cluster orbits; the error bars mark the quantiles 0.15 and 0.85 and the horizontal lines show standard deviation for the density.

normalized frequency, see formula (10). We analyze the events when the CLUSTER satellites crossed the equatorial region at the same L values (about 4.4). Thus, the L dependence of T_0 does not play a role for our observations, and, as is seen from equation (10), T_0 increases with increasing plasma density and it also increases with normalized chorus frequency when $\boldsymbol{x} > 0.215$. These conditions have always been fulfilled for the analyzed chorus elements. For calculation of T_0 from the measured N_e, we use the lower frequency bound of the chorus elements, since T_0 enters the linear growth rate (equation (4)) which determines the initial part of the elements. Thus, defining the cold plasma density N_e from CLUSTER data, we calculated the time scale T_0 for each chorus element.

[19] In turn, df/dt also depends on the density: as shown by *Macúšová et al.* [2010] the median slope of the chorus elements for the analyzed events decreases with increasing average density. Dependence of the frequency sweep rates and the BWO time scale T_0 on the density for all of the chorus elements, both above and below the half-gyrofrequency, is shown in Figure 1a. It is seen that with increasing density from several ones to 10^2 cm^{-3}, T_0 and df/dt behave in the opposite way, i.e., T_0 shows a distinct increase by almost an order of magnitude from 0.013 s to 0.1 s, while df/dt decreases showing a significant spread of values for the same density.

[20] Figure 1b shows median values of the q parameter, depending on the average cold plasma density for the 7 CLUSTER orbits; the error bars mark the quantiles 0.15 and 0.85, and the horizontal lines show the standard deviation for the density. The values of the q parameter for chorus elements with frequencies below and above one-half of the electron gyrofrequency are marked by black and magenta colors, respectively. It is seen that the median values of the parameter q vary in the range 5–30 and show a weak increase with increasing plasma density if we look at the lower and upper chorus bands separately. Significant scatter of q values for each crossing of the equatorial generation region is observed. The q values are visibly higher for the upper band chorus; the dependence of the q parameter on frequency is considered in more detail below in section 3.3.

[21] Next we discuss the relationship between the parameter q and the factors determining it (T_0 and df/dt) with the density for individual orbits. Let us consider two events with low density (N_e = 1–8 cm^{-3}) for the frequencies below (18 April 2002) and above (6 December 2003) half-gyrofrequency and one event with a high density (N_e = 50–110 cm^{-3}) and the frequencies below half-gyrofrequency (21 October 2001). Figure 2 shows T_0, df/dt, and q (from top to bottom) for these events depending on the density. As expected, T_0 (top plots) increases with increasing density but the increase was slight, only about 1.5 times to a maximum of 2 times. The frequency sweep rates of the chorus elements (middle plots) show almost no dependence on the number density within each individual data set.

[22] Note a large spread in df/dt values for each event. An especially large scatter of df/dt (~10 times) was observed for low densities $N_e < 10$ cm^{-3} for both the lower and upper chorus bands. The bottom plots show that a weak dependence of the q parameter on the density can be seen for selected events, especially for events with low density $N_e < 10$ cm^{-3}.

3.3. The Dependence of the *q* Parameter on the Chorus Frequency

[23] Since the q parameter depends on df/dt and T_0, we next consider their relationship with the chorus frequency as shown in Figure 3a, respectively, for the density of $N_e < 10$ cm^{-3} (red) and $N_e > 10$ cm^{-3} (blue). It is seen that the values of df/dt do not depend on frequency for the low densities $N_e < 10$ cm^{-3}, show a slight increase with increasing frequency for high densities $N_e > 10$ cm^{-3}, and have a significant spread of values for all densities and frequencies.

[24] As is seen from Figure 3b the BWO time scale T_0, on the contrary, does not show a significant spread of values at all frequencies, but increases with frequency, especially at higher frequencies $>f_{ce}/2$. Figure 3c shows the values of the parameter q for all chorus elements depending on the frequency; here, the color also marks q values for densities below 10 cm^{-3} (magenta) and above 10 cm^{-3} (blue). It is seen, in spite of the large spread, that the values of q are generally higher for higher plasma density at frequencies below $f_{ce}/2$ and increase with frequency for the band above $f_{ce}/2$.

[25] Now we consider the frequency dependence of the median values of the q parameter. Figure 3d shows the median values of q (q_m) for the intervals $\Delta f/f_{ce} = 0.1$ as a

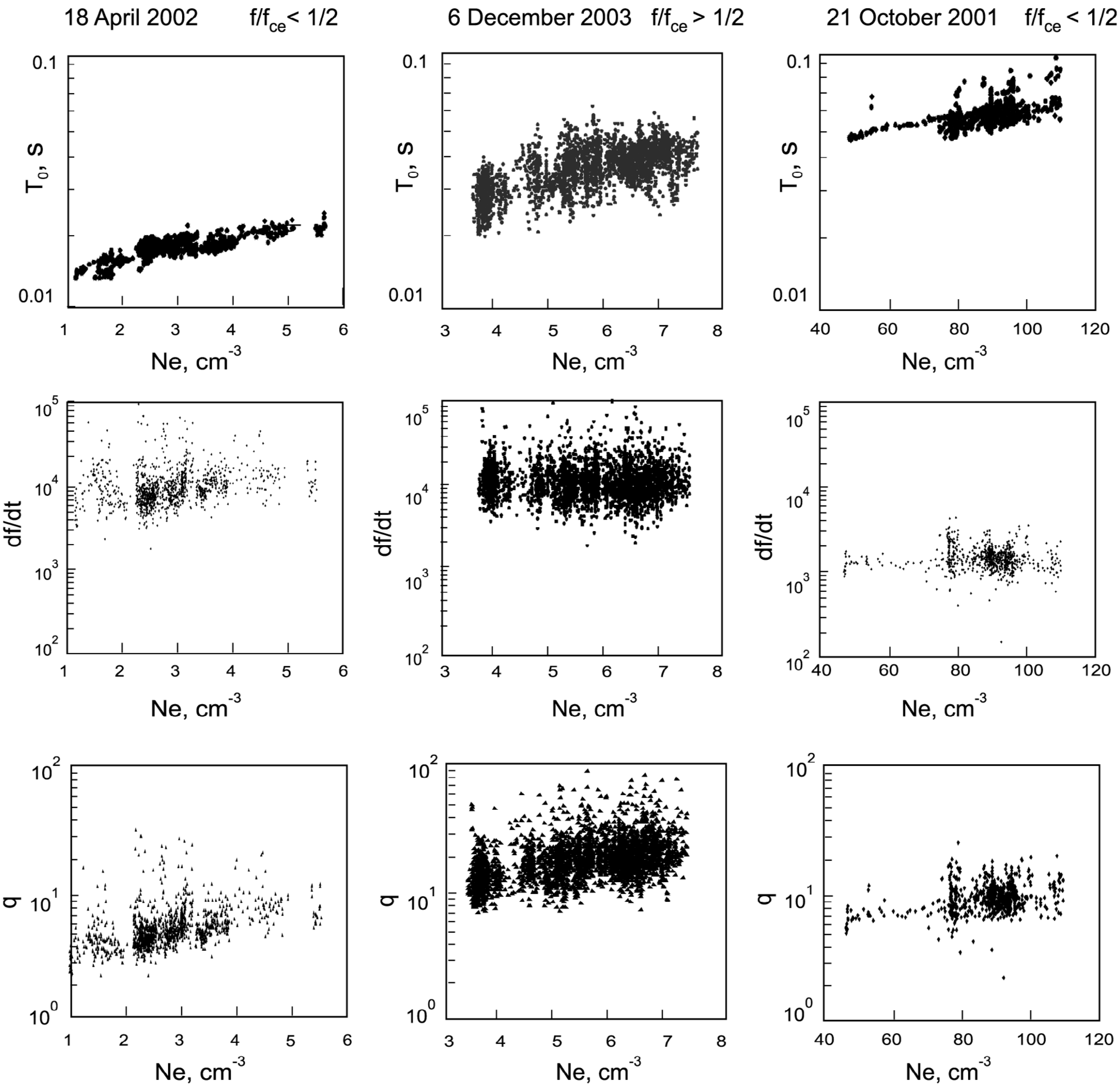

Figure 2. Examples of the BWO time scales T_0, the frequency sweep rates *df*/*dt*, and *q* parameters depending on the density for the three events with the frequencies below (18 April 2002, 21 October 2001) and above (6 December 2003) one-half of the electron gyrofrequency.

function of chorus frequency; the error bars mark quantiles of 0.15 and 0.85. As is seen in Figure 3d, the median values of the *q* parameter are higher for upper band chorus; the median values of the *q* parameter for the lower-band chorus are q_m $(f/f_{ce} < 0.5) \approx 7$ and q_m $(f/f_{ce} > 0.5) \approx 17$ for the upper band; the median *q* parameter for all chorus elements is 13.

[26] It is noteworthy that lower and upper band chorus events are often observed simultaneously, so let us discuss possible explanations for the relationship between the *q* values in this case. According to Polar data analyzed by *Lauben et al.* [2002] and *Haque et al.* [2010], chorus elements observed right near the equator have a predominantly quasi-parallel propagation direction, but in about 50% of the cases, upper-band chorus is detected near the resonance cone even there.

[27] If we assume that both upper and lower band chorus elements are generated at small values of wave normal angles, then it follows from the cyclotron resonance condition that they are produced by electrons with different values of $v_{||}$, i.e., upper-band chorus is generated by electrons with smaller $|v_{||}|$. In this case, we can assume that higher *q* values which characterize higher effective heights of the step, are related to an increase in energetic-electron flux with decreasing energy.

[28] If, on the contrary, both chorus bands are generated by the same region (step) in parallel velocities, then the cyclotron resonance condition implies that their parallel

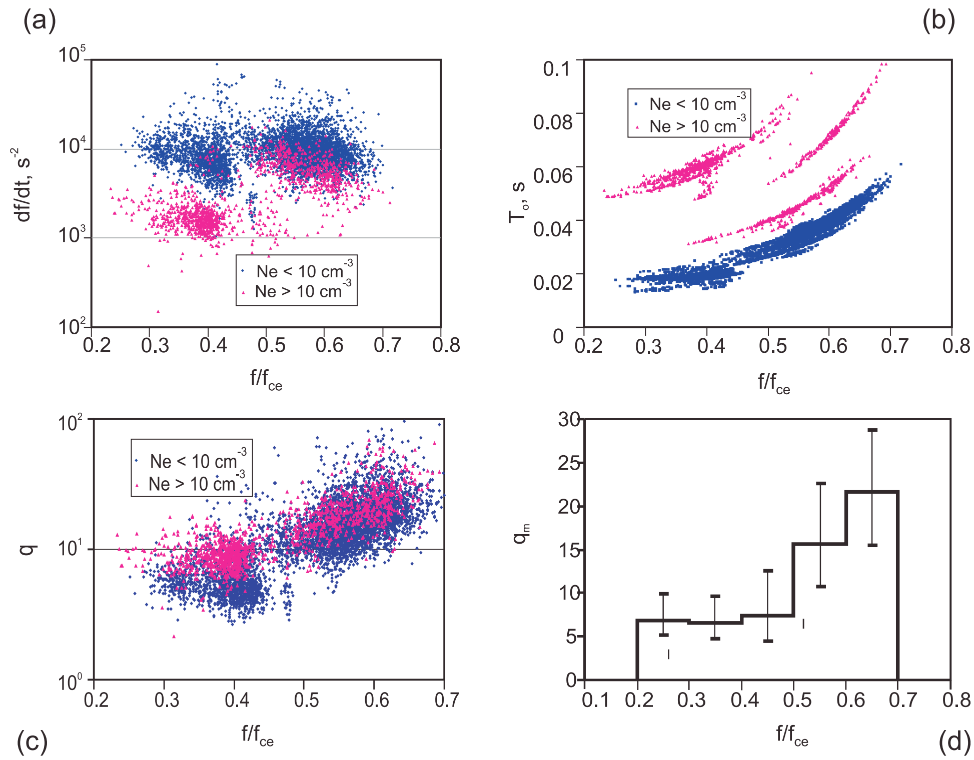

Figure 3. (a) The frequency sweep rate df/dt, (b) the BWO time scale T_0, and (c) the q parameter depending on the chorus frequency normalized to the electron gyrofrequency (f/f_{ce}) for the density of $N_e < 10\ \mathrm{cm}^{-3}$ (magenta) and $N_e > 10\ \mathrm{cm}^{-3}$ (blue). (d) The median values of the parameter q in the frequency intervals 0.1 f/f_H, the error bars marking quantiles 0.15 and 0.85.

wave numbers $k_\|$ and, hence, their wave normal angles, are different. In this case, however, a comparison with the BWO model will require generalizing the model to the case of oblique propagation which has not yet been done. Thus, only preliminary considerations on this matter can be made. The cyclotron-instability growth rate decreases as the wave propagation direction deviates from the parallel because the wave polarization becomes less optimal for the resonant interaction with gyrating electrons [*Kennel*, 1966]. Therefore, we can expect that the actual q value is smaller in the case of oblique propagation than that calculated under the assumption of parallel propagation.

3.4. Distributions and Scatter of the *q* Parameter Values

[29] Significant scatter of the q parameter values was obtained for each CLUSTER satellite crossing through the generation region. This scatter of q values is first of all related to the scatter in the frequency sweep rate of the chorus elements since the BWO time scales T_0 have a small spread; see Figure 1a. The distributions of the q parameter are given in Figures 4a and 4b for the lower ($f/f_{ce} < 0.5$) and upper ($f/f_{ce} > 0.5$) bands of the chorus emissions, respectively, detected by the CLUSTER satellites on 7 orbits. Figure 4 shows that a significant scatter of q parameter

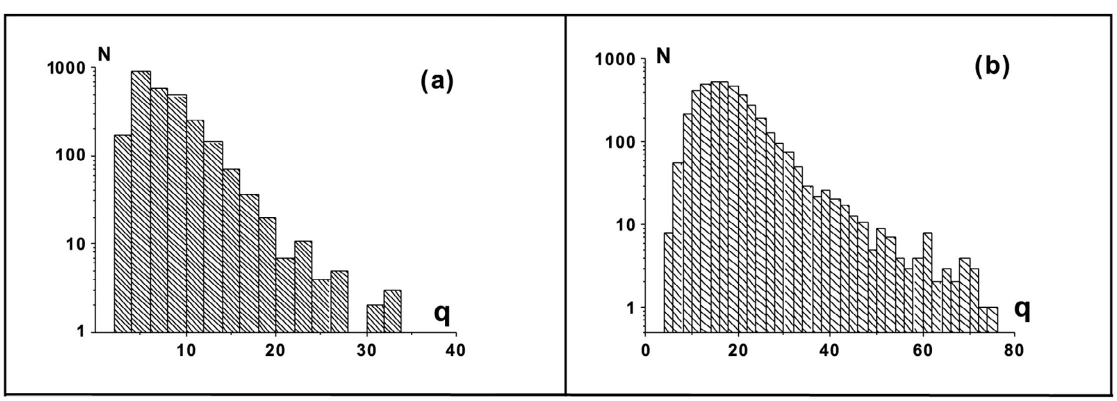

Figure 4. The distribution of the q parameter for chorus elements with frequencies (a) below and (b) above one-half of the electron gyrofrequency for 7 orbits of the CLUSTER spacecraft.

values for both chorus bands is observed, and the shape of the distributions for both bands is similar with a long tail toward higher values where q values may be several times larger than their average. The parameter q is proportional to the relative step height and the energetic electron density (equation (7)), and it also depends on plasma density N_e (via V_g and t_0). We believe that the scatter of q is mostly determined by the scatter in the energetic electron distribution parameters, primarily the relative height b, since other parameters are better known from the experiment, and they do not show such a significant scatter.

4. Discussion

[30] The backward wave oscillator regime of the cyclotron instability was proposed by *Trakhtengerts* [1995, 1999] for generation of chorus emissions in the magnetosphere. The measurements of chorus emissions in the equatorial region by the CLUSTER spacecraft show reasonable agreement with the BWO model. Within the framework of this model, it is possible to explain basic parameters of chorus elements [*Trakhtengerts et al.*, 2004; *Titova et al.*, 2003] and the properties of the chorus source, such as its field aligned scale in the source region and the direction of the energy flux [*Santolík et al.*, 2003, 2004], the motion of the chorus source by deviation of the magnetic field minimum (the local "magnetic equator") [*Kozelov et al.*, 2008]. THEMIS satellite measurements showed that the chorus frequency sweep rate df/dt is proportional to the wave amplitude [*Cully et al.*, 2011] which is consistent with the predictions of both the BWO model and the results obtained by *Omura et al.* [2008].

[31] The sharp gradient (or a step-like deformation) of the electron distribution function obviously is the most important parameter of the BWO model which first of all determines the parameters of the chorus elements. However, the step-like features of the distribution functions have not been observed directly so far which may be due to the fact that the step can be rather small as shown by theoretical estimates based on quasi-linear theory [*Trakhtengerts et al.*, 1996] and preliminary comparison with experiment [*Trakhtengerts et al.*, 2004]. Fortunately, the step-like deformation is related to the measurable parameters of chorus emissions, i.e., to the frequency sweep rate, via the dimensionless parameter q which quantifies the excess of energetic-electron density above the generation threshold.

[32] The q parameter for the magnetospheric BWO was discussed in *Macúšová et al.* [2010] where the relationship between the frequency sweep rate of chorus elements df/dt and the cold plasma density N_e was analyzed on the basis of CLUSTER satellite data. In that paper it was shown that, according to the BWO model, the dependence between df/dt and N_e is given by the following scaling:

$$df/dt \approx \Psi(L, f/f_{ce}, q)\, N_e^{-b} \tag{11}$$

where $b = 2/3$ and Ψ is defined in *Macúšová et al.* [2010, equation (11)]. For CLUSTER observations L values were always near 4.4 and $f/f_{ce} \sim 0.3$ for the lower band and $f/f_{ce} \sim 0.5$ for the upper band correspondingly, the weak dependence of Ψ on f/f_{ce} and L in equation (11) is unimportant for both lower and upper chorus bands taken separately. The most significant unknown in this scaling is the parameter q. *Macúšová et al.* [2010] verified the dependence (11) for the median values of the sweep rates df/dt and the plasma density N_e when the CLUSTER satellites crossed the equatorial chorus generation region. They showed that the sweep rates df/dt indeed decrease with increasing plasma density N_e and $df/dt \sim N_e^{-b}$ where $b = 0.46$. Thus *Macúšová et al.* [2010] get reasonable agreement with theoretical scaling based on the BWO model but their value for b is slightly smaller than expected for q independent of N_e.

[33] This agreement with the theory indicates that the q parameter on average does not change significantly with N_e. However, the spread of the values df/dt is very large for any value of plasma density and in each CLUSTER satellite's crossing of the equatorial chorus generation region; see Figure 1a. Thus, although a rough agreement between the mean values df/dt and N_e was obtained, the authors [*Macúšová et al.*, 2010] have pointed out that the q parameter may vary substantially within the time interval of each event. Therefore, we determined the q parameter values for individual chorus elements and analyzed the relationship of these values of the q parameter with the density and the frequency.

[34] The multi-instrument CLUSTER satellite data allowed us to determine the values of the q parameter in the magnetospheric BWO for chorus elements near the generation region. As it follows from equation (9) the behavior of the parameter q is determined by df/dt and T_0, df/dt is decreasing and T_0 is increasing with N_e (see Figure 1a). As a result, the parameter q depends only weakly on the density of cold plasma; a slight increase of q with increasing density can be seen when comparing different orbits in Figure 1b and sometimes for individual orbits of the CLUSTER spacecraft (Figure 2). This dependence of q on the density corresponds to the observed slower decrease of the sweep rates df/dt with density than $N_e^{-2/3}$ [*Macúšová et al.*, 2010].

[35] One can see in Figure 3c that the values of q increase with frequency and the median values of q obtained for the lower chorus bands is $q_{\text{med}}\,(f/f_{ce} < 0.5) \approx 7$ and for the upper chorus bands is $q_{\text{med}}\,(f/f_{ce} > 0.5) \approx 17$; the median q value for all chorus elements is $q_{\text{med}} \approx 13$. Based on equation (9), large q for the upper band chorus and low density Ne < 10 cm^{-3} is apparently due to the frequency dependence of the characteristic time scale of BWO T_0 (see Figure 3b), since the frequency sweep rate df/dt shows no significant dependence on the frequency for low density. As for possible physical reasons for the difference between q values for the lower and upper bands, we discuss them in section 3.3.

[36] The rather large average values of the q parameter, obtained from the CLUSTER experimental data (see Figure 1b) are in agreement with simulation results by *Demekhov and Trakhtengerts* [2008]. These simulations demonstrated that discrete chorus elements in the BWO regime of the cyclotron instability are formed only for sufficiently large excess above the linear generation threshold, i.e., for $q > 3$. Figure 5 illustrates this result by showing the simulated dynamic spectra based on the BWO model for different values of q. It is seen that for smaller q values ($q \approx 3$), the dynamic spectrum shows only some broadening near the initial generation frequency and possibly some sidebands, whereas discrete elements are clearly seen in the case $q \approx 12$, and they exist also for $q \approx 7$ as shown by *Demekhov and Trakhtengerts* [2008].

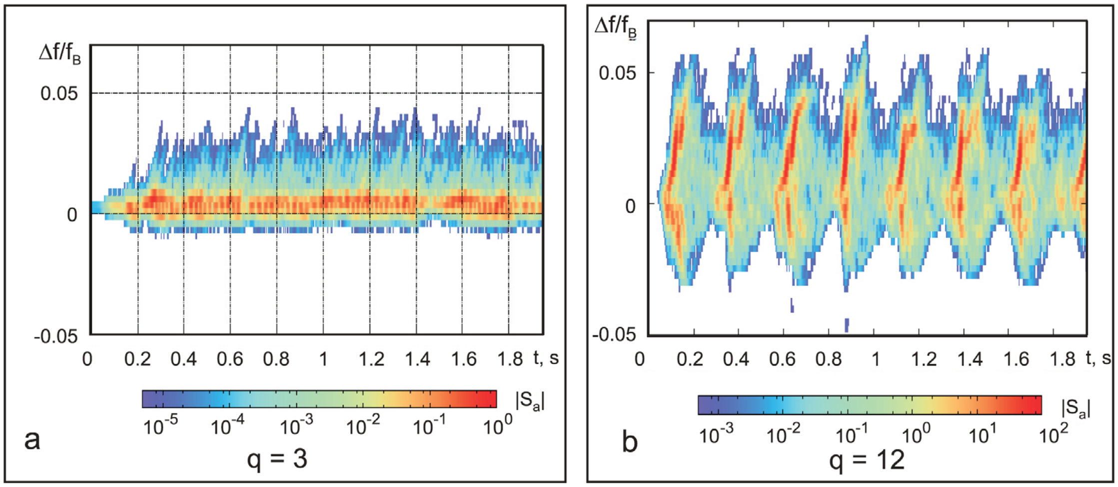

Figure 5. Dynamic spectrums at the exit of the BWO generation region for different values of the q parameter: (a) q = 3, i.e., smaller excess above the threshold for the BWO generation regime, and (b) q = 12, i.e., larger excess. The discrete chorus elements in the BWO regime are formed only for a sufficiently large excess above the linear generation threshold.

[37] Note that recently *Tao et al.* [2012] compared THEMIS data on chorus *df*/*dt* with the BWO model and *Helliwell* [1967] model based on strictly linear approach. They used q = 2 for such comparison and found that df/dt was about 4 times smaller than observed but exhibited the same scaling with plasma density and L shell. Our calculations give q about 7 for Cluster data, which yields much better agreement also with THEMIS data, based on *Tao et al.* [2012] results.

[38] Knowing the values of q we can estimate the relative height of the step b at the distribution function by using equation (8). Let us assume the smooth distribution function to be bi-Maxwellian:

$$F = \frac{1}{\pi^{\frac{3}{2}} \upsilon_{\perp 0}^2 \upsilon_{\parallel 0}} \exp\left(-\frac{\upsilon_\perp^2}{\upsilon_{\perp 0}^2} - \frac{\upsilon_\parallel^2}{\upsilon_{\parallel 0}^2} \right) \quad (12)$$

where $\upsilon_\perp$and $\upsilon_\parallel$ are the characteristic transverse and parallel velocities, respectively, of electrons. Then from equation (8) we obtain

$$b = \frac{\pi^{\frac{3}{2}}}{16} \frac{mc^2}{e^2} \frac{\upsilon_{st}}{l^2} \frac{\upsilon_{\parallel 0}^2}{\upsilon_{\perp 0}^2} \frac{q}{N_h} e^{\upsilon_{st}^2/\upsilon_{\parallel 0}^2} \quad (13)$$

From equation (13) we obtain values of the step in the range of $b \approx 0.01$–0.3 substituting for q-values obtained above and reasonable characteristic plasma parameters such as a transverse anisotropy of the energetic electrons of A = 2 as well as an integral omnidirectional electron flux of $S = (N_h\ \upsilon_{\parallel 0}) \approx 10^8$ 1/(cm^2 s) and $\upsilon_{\parallel 0} \approx V_{st}$, measured in the equatorial region of the chorus source by the CLUSTER satellite [*Trakhtengerts et al.*, 2004; *Kozelov et al.*, 2008]. These b-values are in agreement with the results of *Trakhtengerts et al.* [2004], who assumed b = 0.17 to fit the theory with direct measurements of the chorus growth rate reported by *Santolík et al.* [2003].

[39] The observations of VLF chorus by CLUSTER show that the frequency sweep rate of the chorus elements for each crossing of the equatorial region have a large spread of values, as it is seen in Figures 1a and 2 for both the upper and lower bands of chorus. Note that the significant scatter of values was observed not only for the region of generation as a whole, but sometimes almost at one point where parameters of cold and hot plasma were the same. This scatter of the frequency sweep rate of the chorus elements leads to the scatter in the calculated q values. The statistical distribution of q values is highly asymmetrical with a long tail toward higher values for both the lower and upper chorus bands, which is shown in Figure 4. This asymmetry leads to an excess of the average values of the q parameter by several times compared to the median values. Thus, the values of q, obtained from CLUSTER satellite data, show that the chorus elements can always be generated at a level that significantly exceeds the threshold of the magnetospheric BWO.

[40] A large spread in q values and the variability of the chorus lower frequency bound indicate that both the position of the step on the distribution function and its amplitude are fluctuating. These fluctuations can be related to several factors, including the variability of the underlying noise which forms the step-like distortion, the bounce oscillations of energetic electrons, and magnetic field fluctuations. Note also that the step sharpness can fluctuate as well, which leads to an additional factor for the variability of the effective step height.

[41] *Kozelov et al.* [2003] assumed that a noise-like component exists in the gain of the BWO generator and showed that an ≪on-off≫ intermittency regime can occur in the BWO near the generation threshold in this case. A power law distribution of the time intervals between bursts of wave generation is a manifestation of this intermittency regime. This type of distribution is revealed for time intervals between the chorus elements measured by the CLUSTER satellites [*Trakhtengerts et al.*, 2004] and by a ground based station [*Kozelov et al.* 2003]. Thus, the significant scatter of the q parameter obtained in our study using CLUSTER data

supports the assumption of the importance of fluctuations of the step characteristics.

5. Conclusions

[42] We determined the dimensionless parameter q for the magnetospheric BWO model for generation of chorus VLF emissions. This parameter quantifies an excess of the electron flux over the absolute-instability threshold. According to the BWO theory, the parameter q is related to the step feature in the electron distribution function, and it can be obtained indirectly by measuring the frequency drift rates of chorus elements, total plasma density, and magnetic field (see equations (9)–(10)). Using the measured spectral characteristics of VLF chorus elements detected by the WBD instrument onboard the CLUSTER spacecraft and the electron density obtained from WHISPER data we determined that the median values of the q parameter in the generation region is $q \geq 7$. Note that we do not prove in this work that the used relations based on the BWO model are correct. However, assuming that they are, we can conclude that the chorus elements are formed only when the threshold of the absolute cyclotron instability of whistler mode waves in the near-equatorial region of the magnetosphere is exceeded significantly (by a factor of $q > 7$ in terms of energetic-electron flux). These values of q are in agreement with the results of numerical simulations of chorus elements which are based on the BWO model but which do not use the relations employed in this paper [*Demekhov and Trakhtengerts*, 2008].

[43] Using the obtained values of q we estimated the relative height of the step in the electron distribution function to lie in the range b = 0.01–0.3. This agrees with earlier estimates based on direct measurements of chorus growth rates obtained from CLUSTER data [*Trakhtengerts et al.*, 2004; *Santolík et al.*, 2003].

[44] A significant spread in the q values was observed during each Cluster passage through the generation region. This supports the assumption of the importance of fluctuations in the step characteristics. We believe that further analysis of the q parameter on the basis of CLUSTER wave data along with the data of the flux of energetic electrons will allow us to improve the understanding of electron distribution function dynamics in the chorus generation region.

[45] The BWO model has been developed under the approximation of a vertical step, i.e., the step width in the longitudinal velocities was assumed to be infinitely small. If the step has a finite width, then the efficiency of wave-particle interactions is reduced and, correspondingly, the generation threshold in terms of energetic-electron flux is increased. Analysis of the influence of the step width on the parameters of the BWO regime, as well as more direct verifications of the BWO theory, remain as important tasks for both further theoretical and experimental studies.

[46] **Acknowledgments.** The present work was supported under the GACR grant 205/10/2279, the ME 10001, the grants RFBR 11-02-00654, 11-02-00397, RFBR-CNRS N10-02-9311, and the Program of the Russian Academy of Sciences #22 "Fundamental problems of the Solar system exploration." The University of Iowa acknowledges support from NASA Goddard Space Flight Center grant NNX11AB38G.

[47] Robert Lysak thanks the reviewers for their assistance in evaluating this paper.

References

Balogh, A., et al. (2001), The Cluster magnetic field investigation: Overview of in-flight performance and initial results, *Ann. Geophys.*, *19*, 1207–1217, doi:10.5194/angeo-19-1207-2001.

Bespalov, A. A., and A. G. Demekhov (2009), A linear theory of the backward-wave-oscillator regime in the magnetospheric cyclotron ELF/VLF maser, *Radiophys. Quantum Electron.*, *52*(11), 761–773, doi:10.1007/s11141-010-9186-z.

Bortnik, J., R. M. Thorne, and N. P. Meredith (2008), The unexpected origin of plasmaspheric hiss from discrete chorus emissions, *Nature*, *452*, 62–66, doi:10.1038/nature06741.

Cully, C. M., V. Angelopoulos, U. Auster, J. Bonnell, and O. Le Contel (2011), Observational evidence of the generation mechanism for rising-tone chorus, *Geophys. Res. Lett.*, *38*, L01106, doi:10.1029/2010GL045793.

Décréau, P. M. E., et al. (2001), Early results from the Whisper instrument on Cluster: An overview, *Ann. Geophys.*, *19*, 1241–1258, doi:10.5194/angeo-19-1241-2001.

Demekhov, A. G., and V. Y. Trakhtengerts (2005), Dynamics of the magnetospheric cyclotron ELF/VLF maser in the backward-wave-oscillator regime. I. Basic equations and results in the case of a uniform magnetic field, *Radiophys. Quantum Electron.*, *48*, 639–649, doi:10.1007/s11141-005-0109-3.

Demekhov, A. G., and V. Y. Trakhtengerts (2008), Dynamics of the magnetospheric cyclotron ELF/VLF maser in the backward wave oscillator regime. II. The influence of the magnetic-field inhomogeneity, *Radiophys. Quantum Electron.*, *51*, 880–889, doi:10.1007/s11141-009-9093-3.

Demekhov, A. G., V. Y. Trakhtengerts, M. M. Mogilevsky, and L. M. Zelenyi (2003), Current problems in studies of magnetospheric cyclotron masers and new space project "Resonance," *Adv. Space Res.*, *32*(3), 355–374, doi:10.1016/S0273-1177(03)90274-2.

Demekhov, A. G., V. Y. Trakhtengerts, M. J. Rycroft, and D. Nunn (2009), Efficiency of electron acceleration in the Earth's magnetosphere by whistler-mode waves, *Geomagn. Aeron.*, *49*, 24–29, doi:10.1134/S0016793209010034.

Ginzburg, N. S., and S. P. Kuznetsov (1981), Periodic and stochastic regimes in electron generators with distributed interaction (in Russian), in *Relativistic HF Electronics*, pp. 101–104, Inst. of Appl. Phys., Gorky, Russia.

Gurnett, D. A., et al. (2001), First results from the Cluster wideband plasma wave investigation, *Ann. Geophys.*, *19*, 1259–1272, doi:10.5194/angeo-19-1259-2001.

Haque, N., M. Spasojevic, O. Santolík, and U. S. Inan (2010), Wave normal angles of magnetospheric chorus emissions observed on the Polar spacecraft, *J. Geophys. Res.*, *115*, A00F07, doi:10.1029/2009JA014717.

Helliwell, R. A. (1967), A theory of discrete VLF emissions from the magnetosphere, *J. Geophys. Res.*, *72*(19), 4773–4790, doi:10.1029/JZ072i019p04773.

Hikishima, M., S. Yagitani, Y. Omura, and I. Nagano (2009), Full particle simulation of whistler-mode rising chorus emissions in the magnetosphere, *J. Geophys. Res.*, *114*, A01203, doi:10.1029/2008JA013625.

Horne, R. B., and R. M. Thorne (1998), Potential waves for relativistic electron scattering and stochastic acceleration during magnetic storms, *Geophys. Res. Lett.*, *25*, 3011–3014, doi:10.1029/98GL01002.

Kennel, C. (1966), Low-frequency whistler mode, *Phys. Fluids*, *9*(11), 2190–2202, doi:10.1063/1.1761588.

Kozelov, B. V., E. E. Titova, A. A. Lyubchich, V. Y. Trakhtengerts, and Y. Manninen (2003), On-off intermittency as a possible mechanism of formation of ELF-VLF chorus series, *Geomagn. Aeron.*, *43*(5), 593–601.

Kozelov, B. V., A. G. Demekhov, E. E. Titova, V. Y. Trakhtengerts, O. Santolik, E. Macusova, D. A. Gurnett, and J. S. Pickett (2008), Variations in the chorus source location deduced from fluctuations of the ambient magnetic field: Comparison of Cluster data and the backward wave oscillator model, *J. Geophys. Res.*, *113*, A06216, doi:10.1029/2007JA012886.

Lauben, D. S., U. S. Inan, T. F. Bell, and D. A. Gurnett (2002), Source characteristics of ELF/VLF chorus, *J. Geophys. Res.*, *107*(A12), 1429, doi:10.1029/2000JA003019.

Macúšová, E., et al. (2010), Observations of the relationship between frequency sweep rates of chorus wave packets and plasma density, *J. Geophys. Res.*, *115*, A12257, doi:10.1029/2010JA015468.

Meredith, N. P., M. Cain, R. B. Horne, R. M. Thorne, D. Summers, and R. R. Anderson (2003), Evidence for chorus-driven electron acceleration to relativistic energies from a survey of geomagnetically disturbed periods, *J. Geophys. Res.*, *108*(A6), 1248, doi:10.1029/2002JA009764.

Omura, Y., Y. Katoh, and D. Summers (2008), Theory and simulation of the generation of whistler-mode chorus, *J. Geophys. Res.*, *113*, A04223, doi:10.1029/2007JA012622.

Parrot, M., O. Santolík, D. Gurnett, J. Pickett, and N. Cornilleau-Wehrlin (2004), Characteristics of magnetospherically reflected chorus waves observed by Cluster, *Ann. Geophys.*, *22*, 2597–2606, doi:10.5194/angeo-22-2597-2004.

Santolík, O., D. A. Gurnett, J. S. Pickett, M. Parrot, and N. Cornilleau-Wehrlin (2003), Spatio-temporal structure of storm-time chorus, *J. Geophys. Res.*, *108*(A7), 1278, doi:10.1029/2002JA009791.

Santolík, O., D. A. Gurnett, J. S. Pickett, M. Parrot, and N. Cornilleau-Wehrlin (2004), A microscopic and nanoscopic view of storm-time chorus on 31 March 2001, *Geophys. Res. Lett.*, *31*, L02801, doi:10.1029/2003GL018757.

Santolík, O., J. Chum, M. Parrot, D. A. Gurnett, J. S. Pickett, and N. Cornilleau-Wehrlin (2006), Propagation of whistler mode chorus to low altitudes: Spacecraft observations of structured ELF hiss, *J. Geophys. Res.*, *111*, A10208, doi:10.1029/2005JA011462.

Summers, D., and C. Ma (2000), A model for generating relativistic electrons in the Earth's inner magnetosphere based on gyroresonant wave-particle interactions, *J. Geophys. Res.*, *105*, 2625–2639, doi:10.1029/1999JA900444.

Tao, X., W. Li, J. Bortnik, R. M. Thorne, and V. Angelopoulos (2012), Comparison between theory and observation of the frequency sweep rates of equatorial rising tone chorus, *Geophys. Res. Lett.*, *39*, L08106, doi:10.1029/2012GL051413.

Titova, E. E., B. V. Kozelov, F. Jiricek, J. Smilauer, A. G. Demekhov, and V. Y. Trakhtengerts (2003), Verification of backwards wave oscillator model of VLF chorus generation using data from MAGION 5 satellite, *Ann. Geophys.*, *21*, 1073–1081, doi:10.5194/angeo-21-1073-2003.

Trakhtengerts, V. Y. (1995), Magnetosphere cyclotron maser: Backward wave oscillator generation regime, *J. Geophys. Res.*, *100*, 17,205–17,210, doi:10.1029/95JA00843.

Trakhtengerts, V. Y. (1999), A generation mechanism for chorus emission, *Ann. Geophys.*, *17*, 95–100.

Trakhtengerts, V. Y., M. J. Rycroft, and A. G. Demekhov (1996), Interrelation of noise-like and discrete ELF/VLF emissions generated by cyclotron interactions, *J. Geophys. Res.*, *101*, 13,293–13,301, doi:10.1029/95JA03515.

Trakhtengerts, V. Y., A. G. Demekhov, E. E. Titova, B. V. Kozelov, O. Santolik, D. Gurnett, and M. Parrot (2004), Interpretation of cluster data on chorus emissions using the backward wave oscillator model, *Phys. Plasmas*, *11*, 1345–1351, doi:10.1063/1.1667495.

Tsurutani, B. T., and E. J. Smith (1977), Two types of magnetospheric ELF chorus and their substorm dependences, *J. Geophys. Res.*, *82*, 5112–5128, doi:10.1029/JA082i032p05112.